

Theorem 2.1 (Pythagoras [7]).

Let a, b, c denote the sides of a triangle. If the angle between a, b equals 90° then

$$|a|^2 + |b|^2 = |c|^2.$$

Moreover, regardless of the assumption on the angle, it holds that $|a| + |b| \geq |c|$.

See References for the bibliography style in CEJM. Below is a proposition with a proof.

Proposition 2.1.

The only possible real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a},$$

where $\Delta = b^2 - 4ac$.

Proof. Observe that

$$(x - x_1)(x - x_2) = -\frac{\Delta}{4a^2} + x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} \tag{1}$$

Taking into account that $\Delta = b^2 - 4ac$, equation (1) leads to

$$(x - x_1)(x - x_2) = \frac{c}{a} + x^2 + \frac{bx}{a} \tag{2}$$

Finally, (2) shows that $a(x - x_1)(x - x_2)$ equals the original equation. A polynomial of degree two cannot have more than two roots, therefore x_1, x_2 are the only possible solutions of our equation. \square

Corollary 2.1.

If $b^2 < 4ac$ then the quadratic equation $ax^2 + bx + c = 0$ has no real solutions.

2.1. Subsections and sample formulas

This is a subsection.

Typical aligned list of formulas with labels, displayed by using the `alignenvironment`.

$$(b + a)(2b - 3a) = 2b^2 - ab - 3a^2 \tag{3}$$

$$123 + 321 = 444 \tag{4}$$

The same list without labels.

$$(b + a)(2b - 3a) = 2b^2 - ab - 3a^2$$

$$123 + 321 = 444$$

Now the same list with custom tags.

$$(b + a) (2b - 3a) = 2b^2 - ab - 3a^2 \quad (**)$$

$$123 + 321 = 444 \quad (\dagger)$$

Another example of aligned formulas

$$1 + 1 = 2$$

$$1 + 2 = 3$$

$$1 + 3 = 4$$

with some text inserted between,

$$10 + 1 = 11$$

$$10 + 2 = 12$$

$$10 + 3 = 13$$

$$100 + 1 = 101$$

$$100 + 2 = 102$$

$$100 + 3 = 103$$

by using the `\intertext` command.

2.1.1. Polynomials of degree three

This is a “subsubsection”.

Below is a complicated formula, which must be divided into several lines using, for instance, the `align*` environment:

$$\begin{aligned} x_1 &= \left(-\frac{\sqrt{3}i}{2} - \frac{1}{2} \right) \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}} \\ &\quad + \frac{\left(\frac{\sqrt{3}i}{2} - \frac{1}{2} \right) (a^2 - 3)}{9 \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}}} - \frac{a}{3}, \\ x_2 &= \left(\frac{\sqrt{3}i}{2} - \frac{1}{2} \right) \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}} \\ &\quad + \frac{\left(-\frac{\sqrt{3}i}{2} - \frac{1}{2} \right) (a^2 - 3)}{9 \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}}} - \frac{a}{3}, \\ x_3 &= \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}} \\ &\quad + \frac{a^2 - 3}{9 \left(\frac{3^{-\frac{3}{2}} \sqrt{12a^4 - 12a^3 + 188a^2 - 432a + 247}}{2} - \frac{2a^3 + 72a - 81}{54} \right)^{\frac{1}{3}}} - \frac{a}{3} \end{aligned}$$

Theorem 2.2.

The numbers x_1, x_2, x_3 defined above are the only complex solutions of the equation

$$x^3 + ax^2 + x + 3(a - 1) = 0.$$

Proof. Easy calculations, using `Maxima` or some other free computer algebra system. □

Table 1. Some caption text.

<i>Some title</i>			
row 1, column 1	row 1, column 2		
row 2, column 1	row 2, column 2		
row 3, column 1	row 3, column 2		
<i>Another title</i>	Value 1	Value 2	Value 3
row 1	130	30	30
row 2	1025	1	15
row 3	100	1	10
row 4	2925	1	4
row 5	2950	1	2

Other polynomials

This is a paragraph with title.

One should admit that there is no general formula for the solutions of general polynomial equations and therefore sometimes we must use numerical methods.

Example 2.1.

Consider the polynomial $v(t) = t^3 + t + 1$ of degree three. It turns out that its only real root is

$$t = \left(\frac{3^{-\frac{3}{2}} \sqrt{31}}{2} - \frac{1}{2} \right)^{\frac{1}{3}} - \frac{1}{3 \left(\frac{3^{-\frac{3}{2}} \sqrt{31}}{2} - \frac{1}{2} \right)^{\frac{1}{3}}}$$

There are also other two complex roots¹, which we ignore.

Remark 2.1.

The proof of Theorem 2.2 uses some advanced technology, including a computer algebra system. The same applies to Example 2.1. On the other hand, Proposition 2.1 is well known and its proof is rather elementary.

3. Tables and figures

Table 1 shows how to show some data using the `table` environment.

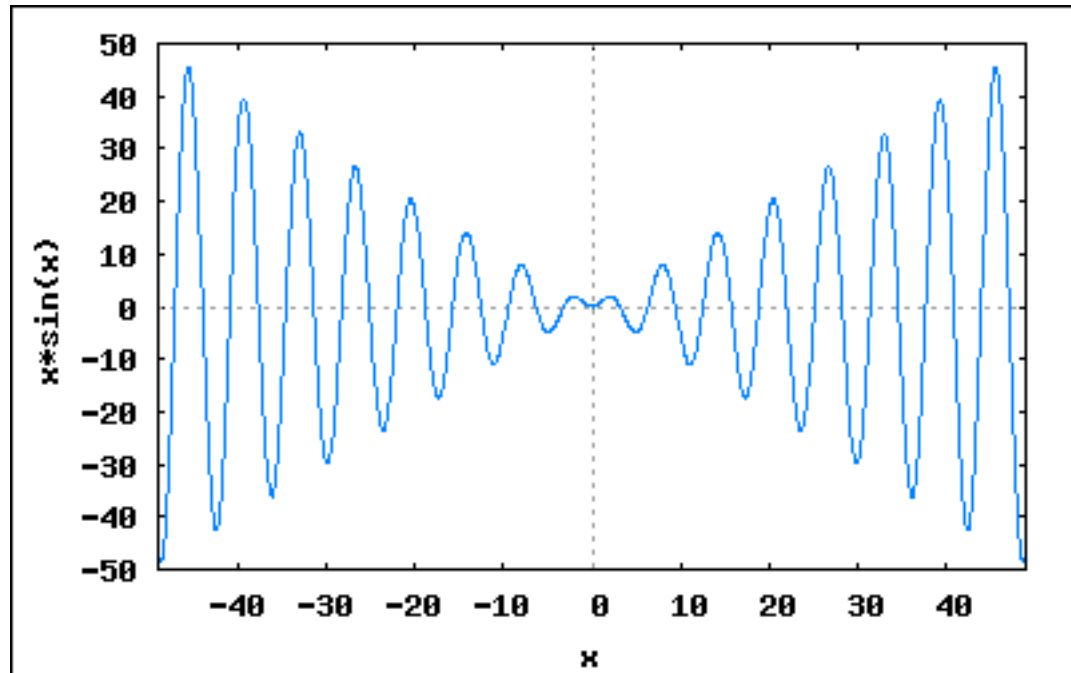
Figure 1 shows how to use the `figure` environment for displaying graphics, etc.

Acknowledgements

The author(s) would like to thank some institutions for support and so on.

¹ Since the degree of v is 3, we know that there may be at most 3 roots.

Figure 1. The graph of $y = x \sin x$ in the interval $[-50,50]$, created by wxMaxima 0.7.4.



References

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